4.4 Irrational Numbers

Part I: *Converting Radicals to Powers with Rational Exponents and Vice Versa*

Label....

$$\sqrt[n]{x}$$

n = index

$\sqrt{}$ = radical

x = radicand

A power can be expressed as a radical and vice-versa via:

$$\boxed{x^{\frac{1}{n}} = \sqrt[n]{x} \quad \underline{\underline{\text{and}}} \quad x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad \underline{\text{or}} \quad \left(\sqrt[n]{x}\right)^m}$$

Convert from a Power to a Radical

$(8)^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

$9^{-\frac{3}{2}} = \dfrac{1}{9^{\frac{3}{2}}} = \dfrac{1}{\sqrt{9^3}} = \dfrac{1}{\sqrt{729}} = \dfrac{1}{27}$

(Each radicand is a number *so CAN be evaluated.*

$(x)^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$ $\underline{\underline{\text{or}}}$ $\sqrt[3]{x^2}$

$(xy)^{-\frac{1}{2}} = \dfrac{1}{xy^{\frac{1}{2}}} = \dfrac{1}{\sqrt{xy}}$

(Each radicand is a variable *so CANNOT be evaluated, but only simplified.*

Convert from a Radical to a Power

$\sqrt{xy} = xy^{\frac{1}{2}}$ $\sqrt[3]{b^2} = b^{\frac{2}{3}}$

$\dfrac{1}{\sqrt[5]{(2x)^3}} = \dfrac{1}{(2x)^{\frac{3}{5}}} = (2x)^{-\frac{3}{5}}$ $\dfrac{1}{4\sqrt[5]{x^7}} = \dfrac{1}{4x^{\frac{7}{5}}} = \dfrac{x^{-\frac{7}{5}}}{4}$

note the brackets for the base

note no brackets, the base does not include $\frac{1}{4}$

Part II: Converting from an Entire Radical to a Mixed Radical and Vice Versa

Rewrite $\sqrt{24}$ as a product of two square roots, one of which has a radicand that is a ***perfect square***. Then simplify.

$\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

$\sqrt{24}$ is called an **entire radical.**
$2\sqrt{6}$ is called a **mixed radical.**

Simplify $\sqrt[3]{24}$. Start by rewriting $\sqrt[3]{24}$ as a product of two cube roots, one of which has a radicand that is a perfect cube.

$$\sqrt[3]{24} = \sqrt[3]{8 \times 4} = \sqrt[3]{8}\,\sqrt[3]{4} = 2\sqrt[3]{4}$$

Can $\sqrt[4]{24}$ be simplified using the same procedure? Explain.

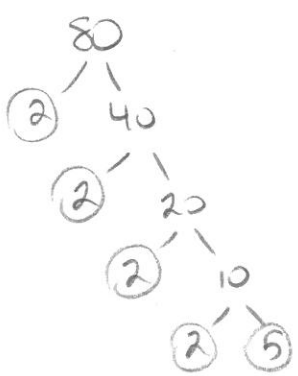
$\sqrt[4]{24}$

the smallest quartic root is 2 for the number $2^4 = 16$ but 16 is not a factor of 24. Since the smallest possible power of 4 is not a factor we cannot simplify $\sqrt[4]{24}$

We can also use prime factorization to simplify a radical.

To do this, start by writing the radicand as a product of prime factors, then simplify.

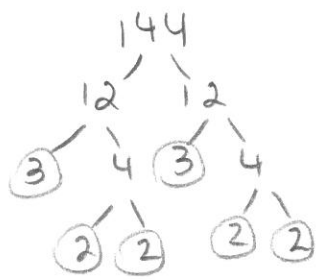
$\sqrt{80}$



$$\sqrt{80} = \sqrt{\underbrace{2\times2\times2\times2}_{2^4}\times5}$$

$$\sqrt{80} = 2\sqrt{5}$$

$\sqrt[3]{144}$

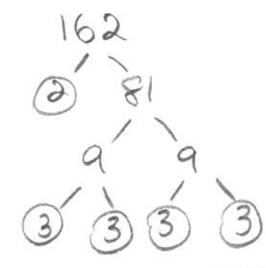


$$\sqrt[3]{144} = \sqrt{3\times3\times2\times\underbrace{2\times2\times2}_{2^3}}$$

$$\sqrt[3]{144} = 2\sqrt[3]{3\times3\times2}$$

$$\sqrt[3]{144} = 2\sqrt[3]{18}$$

$\sqrt[4]{162}$



$$\sqrt[4]{162} = \sqrt[4]{2\times\underbrace{3\times3\times3\times3}_{3^4}}$$

$$\sqrt[4]{162} = 3\sqrt[4]{2}$$

an ***Entire Radical*** is a radical of the form:

$\sqrt[n]{x}$ where $x \in \mathbb{Z}$, $n \in \mathbb{Z}_+$

a ***Mixed Radical*** is a radical of the form: integers whole numbers

$a\sqrt[n]{b}$ where $a, b \in \mathbb{Z}$, $n \in \mathbb{Z}_+$

Convert Entire Radicals to Mixed Radicals

$\sqrt{45}$
$= \sqrt{9 \times 5}$
$= \sqrt{9}\sqrt{5}$
$= 3\sqrt{5}$

$\sqrt{75}$
$= \sqrt{25 \times 3}$
$= \sqrt{25}\sqrt{3}$
$= 5\sqrt{3}$

$\sqrt{80}$
$= \sqrt{16 \times 5}$
$= \sqrt{16}\sqrt{5}$
$= 4\sqrt{5}$

$\sqrt{98}$
$= \sqrt{49 \times 2}$
$= \sqrt{49}\sqrt{2}$
$= 7\sqrt{2}$

$\sqrt[3]{32}$
$= \sqrt[3]{8 \times 4}$
$= \sqrt[3]{8}\sqrt[3]{4}$
$= 2\sqrt[3]{4}$

$\sqrt[3]{250}$
$= \sqrt[3]{125 \times 20}$
$= \sqrt[3]{125}\sqrt[3]{20}$
$= 5\sqrt[3]{20}$

$\sqrt{20a^2b^4}$
$= \sqrt{4 \times 5 \times a^2 \times b^2 \times b^2}$
$= \sqrt{4}\sqrt{5}\sqrt{a^2}\sqrt{b^2}\sqrt{b^2}$
$= 2\sqrt{5}\,abb$
$= 2ab^2\sqrt{5}$

$\sqrt{50a^2b^2}$
$= \sqrt{25 \times 2 \times a^2 \times b^2}$
$= \sqrt{25}\sqrt{2}\sqrt{a^2}\sqrt{b^2}$
$= 5\sqrt{2}\,a\,b$
$= 5ab\sqrt{2}$

$\sqrt[4]{162}$
$= \sqrt[4]{81 \times 2}$
$= \sqrt[4]{81}\sqrt[4]{2}$
$= 3\sqrt[4]{2}$

Simplest form is usually preferred and often specifically required on tests which is **MIXED RADICAL** form.

Convert Mixed Radicals to Entire Radicals

Write $4\sqrt{3}$ as an entire radical.

$4\sqrt{3} = \sqrt{4\times4}\sqrt{3} = \sqrt{16}\sqrt{3} = \sqrt{16\times3} = \sqrt{48}$

Write $3\sqrt[3]{2}$ as an entire radical.

$3\sqrt[3]{2} = \sqrt[3]{3\times3\times3}\sqrt[3]{2} = \sqrt[3]{27}\sqrt[3]{2} = \sqrt[3]{27\times2} = \sqrt[3]{54}$

Write $2\sqrt[5]{2}$ as an entire radical.

$2\sqrt[5]{2} = \sqrt[5]{2\times2\times2\times2\times2}\sqrt[5]{2} = \sqrt[5]{32}\sqrt[5]{2} = \sqrt[5]{32\times2} = \sqrt[5]{64}$

Try: Convert the following mixed radicals into entire radicals.

$5\sqrt{7}$

$=\sqrt{5\times5}\sqrt{7}$

$=\sqrt{25}\sqrt{7}$

$=\sqrt{25\times7}$

$=\sqrt{175}$

$\frac{1}{2}\sqrt{10}$

$=\sqrt{\frac{1}{2}\times\frac{1}{2}}\sqrt{10}$

$=\sqrt{\frac{1}{4}}\sqrt{10}$

$=\sqrt{\frac{1}{4}\times10}$

$=\sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}}$

$4.2\sqrt{18}$

$=\sqrt{4.2\times4.2}\sqrt{18}$

$=\sqrt{17.64}\sqrt{18}$

$=\sqrt{17.64\times18}$

$=\sqrt{317.52}$

$2\sqrt[3]{5}$

$=\sqrt[3]{2\times2\times2}\sqrt[3]{5}$

$=\sqrt[3]{8}\sqrt[3]{5}$

$=\sqrt[3]{8\times5}$

$=\sqrt[3]{40}$

How would rewriting mixed radicals as entire radicals help you to order a set of mixed radicals with the same index?

You would be able to put them in order by the radicand if they were entire.

Ordering Irrational Numbers

To order radicals WITHOUT a calculator . . .

Put into entire form, order by radicand (if they have the same index)

To order radicals WITH a calculator . . .

Solve for the value of the radical.

Try: Arrange in order from least to greatest **without** the use of a calculator

$3\sqrt{5}$, 10, $2\sqrt{7}$, $4\sqrt{2}$

$3\sqrt{5} = \sqrt{3\times3\times5} = \sqrt{45}$

$10 = 10\sqrt{1} = \sqrt{10\times10} = \sqrt{100}$

$2\sqrt{7} = \sqrt{2\times2\times7} = \sqrt{28}$

$4\sqrt{2} = \sqrt{4\times4\times2} = \sqrt{32}$.

order: $\sqrt{28}, \sqrt{32}, \sqrt{45}, \sqrt{100}$

$2\sqrt{7}, 4\sqrt{2}, 3\sqrt{5}, 10$

Try: Arrange in order from least to greatest **with** the use of a calculator

$2\sqrt[3]{5}$, $\sqrt[3]{13}$, $7\sqrt[3]{2}$, $6\sqrt[3]{3}$

$2\sqrt[3]{5} = 3.42$

$\sqrt[3]{13} = 2.35$

$7\sqrt[3]{2} = 8.82$

$6\sqrt[3]{3} = 8.65$

order: 2.35, 3.42, 8.65, 8.82

$\sqrt[3]{13}, 2\sqrt[3]{5}, 6\sqrt[3]{3}, 7\sqrt[3]{2}$

Part III: *Classifying Numbers*

What is an *irrational* number?

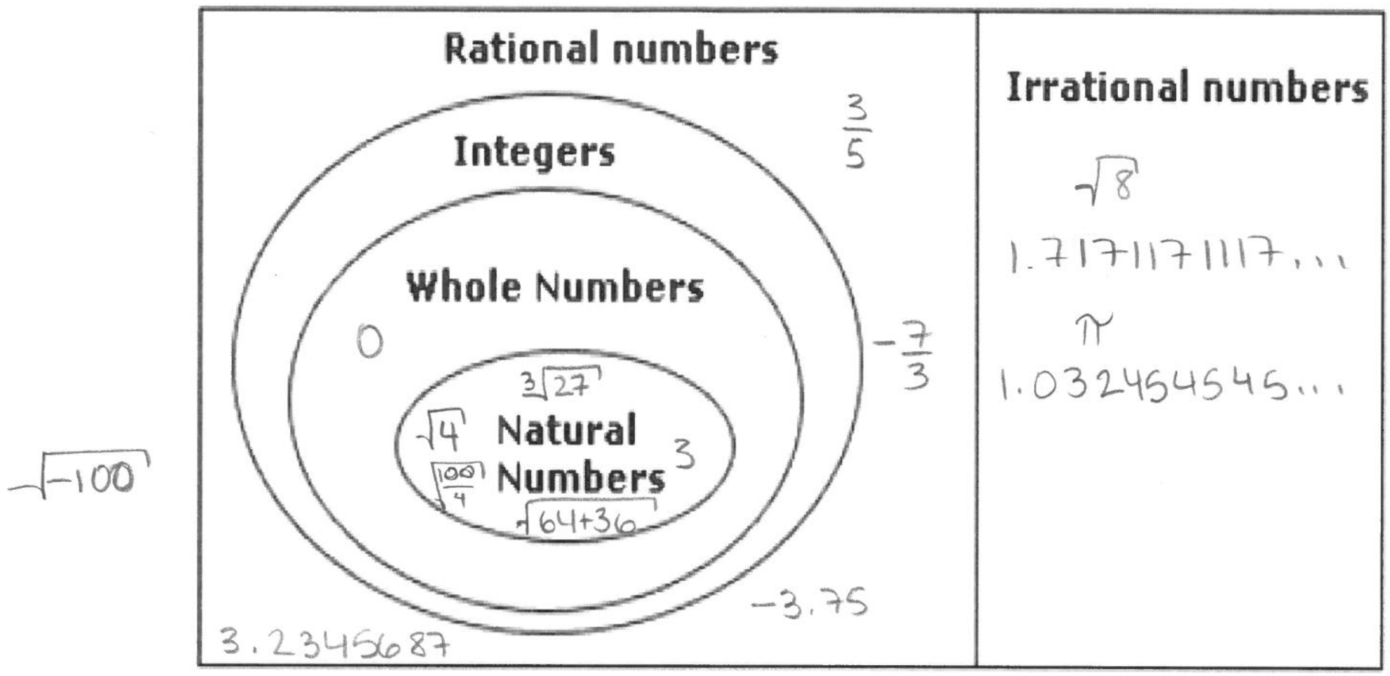
An irrational number cannot be written as a fraction

What is a *rational* number?

A rational number can be written as a fraction.

Rational + Irrational numbers = real numbers

The Real Number System



Try: Classify the following numbers into the correct number system.

(a) $\sqrt{4}$ = 2
Natural

(b) 3 natural

(c) 1.7171171117…
irrational

(d) $\frac{3}{5}$ rational

(e) $\frac{-7}{3}$ rational

(f) $\sqrt{-100}$
complex or imaginary

(g) $\sqrt{8} = 2\sqrt{2}$
irrational

(h) 0 whole

(i) π
irrational

(j) -3.75 $= -\frac{375}{100}$
rational

(k) $\sqrt[3]{27} = 3$ natural

(l) 1.032454545…
irrational

(m) 3.2345687
$= \frac{32345687}{10000000}$
rational

(n) $\sqrt{64+36}$
$= \sqrt{100} = 10$
natural

(o) $\sqrt{\frac{100}{4}} = \frac{\sqrt{100}}{\sqrt{4}} = \frac{10}{2} = 5$
natural

Application:

The relationship between the length and mass of Pacific Halibut can be approximated by $l = 0.46\sqrt[3]{m}$, where l is length in meters and m is mass in kilograms. What is the length of a 25 kg halibut?

$l = 0.46\sqrt[3]{m}$

let $m = 25$ kg

$l = 0.46\sqrt[3]{25}$

$l = (0.46)(2.92\ldots)$

$l = 1.35$ meters.

Homework: p192 #1-7 (def), 8-10, 12